

# SPACECRAFT MOMENTUM MANAGEMENT PROCEDURES\*

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## ABSTRACT

This paper describes techniques appropriate for implementation onboard the Space Telescope and other spacecraft to manage the accumulation of momentum in reaction wheel control systems using magnetic torquing coils. Generalized analytical equations are derived for momentum control laws that command the magnetic torquers. These control laws naturally fall into two main categories according to the methods used for updating the magnetic dipole command: closed-loop, in which the update is based on current measurements to achieve a desired torque instantaneously, and open-loop, in which the update is based on predicted information to achieve a desired momentum at the end of a period of time. Each control law is further categorized by the physical quantities (e.g., energy, wheel speed, etc.) selected for minimization. Physical interpretations of control laws in general and of the Space Telescope cross product and minimum energy control laws in particular are presented, and their merits and drawbacks are discussed. A new technique is introduced to retain the advantages of

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both the open-loop and the closed-loop control laws. Simulation results are presented to compare the performance of these control laws in the Space Telescope environment. The results discussed in the paper can be extended to the Multi-mission Modular Spacecraft (MMS) series and similar missions.

## INTRODUCTION

The Space Telescope (ST) is an astronomical observatory to be launched in 1984 by the Space Shuttle into a nominal 500-kilometer circular orbit. The Pointing Control System provides the attitude reference and control stability for the ST. The most challenging requirement of the Pointing Control System is the pointing stability of 0.007 arc-second (one sigma). To achieve this stability required in the fine point mode, vibrations generated by the rotating reaction wheels must not excite significant ST bending modes.

Two momentum management control laws have been proposed by Lockheed Missiles & Space Company (LMSC) for desaturating the ST reaction wheels, namely, the cross product (CP) control law and the minimum energy (ME) control law. The CP law is a closed-loop control law that computes a control magnetic dipole based on current measurements to achieve a desired torque instantaneously. The ME law is an open-loop control law that generates the magnetic dipole commands based on predicted information to achieve a desired momentum at the end of a period of time, and at the same time minimizes the energy consumption by the magnetic coils. More detailed descriptions of ST momentum management procedures are given in Reference 1 and 2.

To further understand and compare these control laws, we have derived generalized analytical equations for spacecraft momentum management using magnetic torquers and have studied

their physical interpretations. As a result of this study, a new technique, referred to as the "mixed-mode" control law, has been introduced to retain the advantages of both closed-loop and open-loop control laws. The momentum management procedures during maneuvers were also investigated for the original technique and for the new technique. To support the current study, a simulator has been implemented to enable quantitative comparison of the performances of various control laws.

In this paper, the generalized analytical equations are presented first and interpreted. Then the merits and drawbacks of each type of control law are discussed and the basis for the new mixed-mode technique is introduced. The CP and ME laws currently implemented for ST are then described and discussed as special cases. Finally, the expected advantages of the mixed-mode control law over the current CP and ME laws for ST are summarized. The simulation results are not included in this paper because they have not been completely analyzed at this time. However, the simulation results are anticipated to be presented in the Symposium.

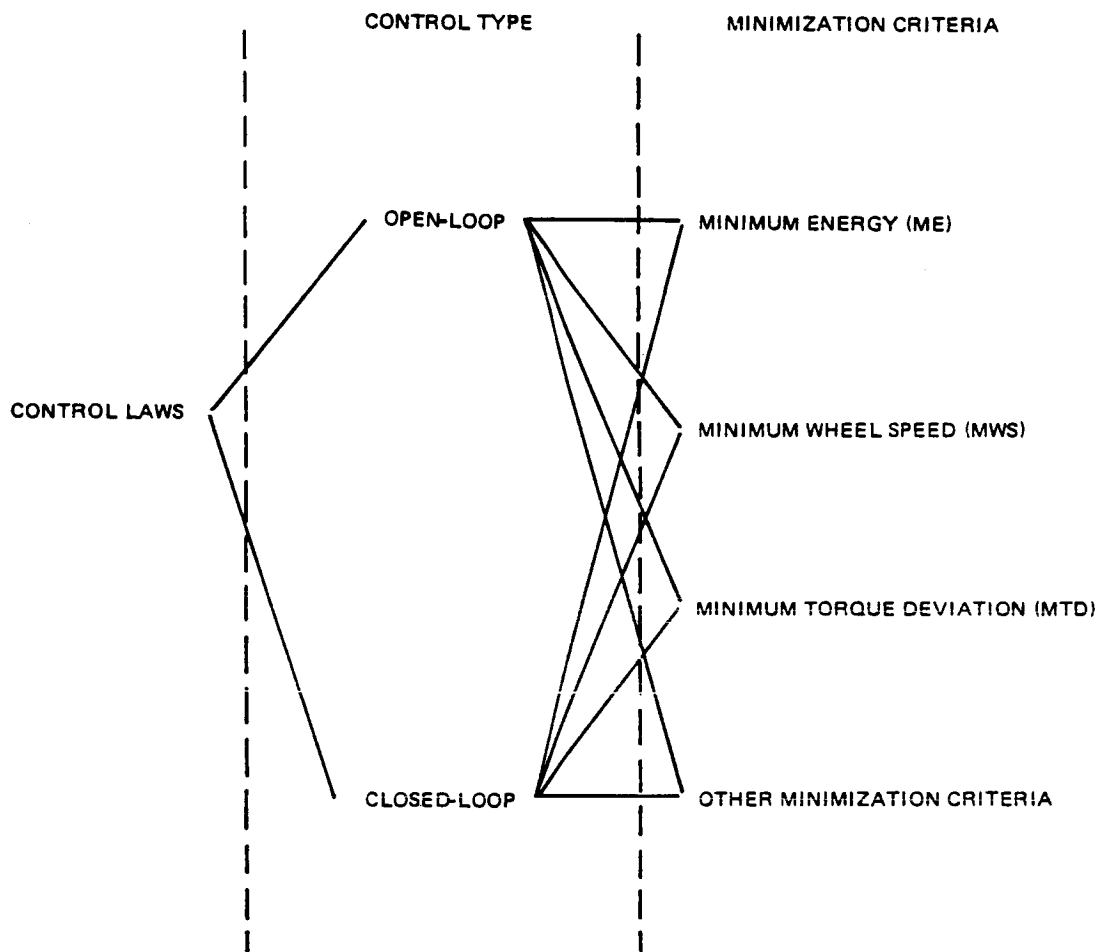
#### GENERALIZED ANALYTICAL EQUATIONS AND PHYSICAL INTERPRETATIONS

In general, a desaturation control law is a method of reducing the buildup of spacecraft momentum due to external environmental torques by generating a magnetic torque resulting from the interaction between the geomagnetic field and the commanded magnetic torques situated on the spacecraft.

There are two fundamental distinctions that characterize a control law: the type of control and the minimization criterion. Each control law can in general be put into one

of the two main categories, depending on its type of control--closed loop or open loop. In a closed-loop control law, the magnetic dipole command is updated using instantaneous measurements with the intent to achieve a desired torque at each update time. In an open-loop control law, the magnetic dipole command is updated using predicted information with the intent to achieve a desired momentum at the end of each update period. In addition to these fundamental differences, the control laws can be further categorized by their minimization criteria. To achieve a desired torque or momentum, there is usually one degree of freedom in commanding the magnetic torquers. This degree of freedom can be used to select one quantity to minimize, such as the energy consumption or the reaction wheel speed.

The minimization criterion is completely independent of the control type. That is, every control law can be either closed loop or open loop regardless of which quantity is minimized. This categorization of control laws is illustrated in Figure 1. Thus, to specify a control law clearly, it is necessary to specify not only the minimization criterion but also the control type. In principle, a minimum energy law can be either a closed-loop law or an open-loop law depending on how the magnetic dipole command is generated. The ST tradition of using "CP law" to represent a closed-loop law and "ME law" to represent an open-loop law is confusing from a physical point of view. In the remainder of this paper, a control law is defined by specifying its control type followed by its minimization criterion, e.g., "closed-loop ME law" or "open-loop minimum wheel speed law." When a particular control law implemented for ST is referred to, the word "original" or "current" will be used to distinguish it from other control laws. For instance, the "current ME law" represents the ME law



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Figure 1. Categorization of Control Laws

implemented currently for ST, which actually is an open-loop ME law.

Table 1 presents the generalized analytical equations for all control laws using magnetic torquers. In the table,  $\vec{T}_D$  is the desired torque, which is the torque a closed-loop control law is attempting to achieve momentarily through the interaction between the magnetic torquers and the geomagnetic field. Here  $\vec{H}_D$ , which is defined for open-loop control laws only, is the integration of the desired torque over a period of time (called the desaturation period). Physically,  $\vec{H}_D$  is the desired momentum an open-loop control law attempts to achieve over the desaturation period through the interaction between the magnetic torquers and the geomagnetic field. Thus, the fundamental difference between the open-loop and the closed-loop control laws is that the former attempts to achieve  $\vec{T}_D$  momentarily, whereas the latter attempts to achieve  $\vec{H}_D$  over a desaturation period. The determination of  $\vec{T}_D$  and  $\vec{H}_D$  depends on the individual control law. However, good momentum management relies on proper determination of  $\vec{T}_D$  and  $\vec{H}_D$ . One reasonable way of defining  $\vec{T}_D$  and  $\vec{H}_D$  is to assume that the gravity-gradient torque  $\vec{T}_{GG}$  is the only significant environmental torque acting on the spacecraft. In this case,

$$\vec{T}_D = \dot{\vec{H}}_T - \vec{T}_{GG} \quad (1)$$

where  $\vec{H}_T$  is the total system momentum which equals the reaction wheel momentum  $\vec{H}_{RW}$  at inertial attitudes. For a closed-loop control law,  $\vec{H}_T$  in Equation (1) is usually replaced by  $-K_M(\vec{H}_T + \vec{H}_B)$ , where  $K_M$  is a positive constant called the magnetic gain and  $\vec{H}_B$  is a bias vector that is added to  $\vec{H}_T$  to keep the reaction wheel speed center at zero. For an open-loop control law, Equation (1) is

Table 1. General Equations for All Control Laws

	CLOSED-LOOP LAW	OPEN-LOOP LAW
DESIRED TORQUE	$\vec{T}_D$	
DESIRED MOMENTUM	NOT APPLICABLE	$\vec{H}_D = \int_{t_i}^{t_f} \vec{T}_D dt$
WEIGHTING MATRIX	A	
COSTATE VECTOR	$\vec{P} = (\vec{B}^T A A^T \vec{B})^{-1} \vec{T}_D$	$\vec{P} = \left[ \int_{t_i}^{t_f} \vec{B}^T A A^T \vec{B} dt \right]^{-1} \vec{H}_D$
SYSTEM MAGNETIC DIPOLE MOMENT	$\vec{\mu}_M = A A^T \vec{B} \vec{P}$	
MAGNETIC TORQUE	$\vec{T}_M = \vec{\mu}_M \times \vec{B}$	
COMMANDED MAGNETIC DIPOLE MOMENT	$\vec{\mu}_T = M^T (MM^T)^{-1} \vec{\mu}_M$	

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integrated over the desaturation period to give the desired momentum  $\vec{H}_D$ . That is,

$$\begin{aligned}\vec{H}_D &= \int_{t_i}^{t_f} \vec{T}_D dt \\ &= \vec{H}_T(t_f) - \vec{H}_T(t_i) - \int_{t_i}^{t_f} \vec{T}_{GG} dt\end{aligned}\tag{2}$$

where  $\vec{H}_T(t_f)$  is the desired total momentum at the end of the desaturation period and  $\vec{H}_T(t_i)$  is the measured total momentum at the start of the desaturation period. The length of the desaturation period controls the magnitude and direction of  $\vec{H}_D$ . Nominally, the desaturation period is set at half an orbital period to include the major variations in the geomagnetic field and to be compatible with the period of the gravity-gradient disturbances so that only the nonperiodic portion of the accumulated gravity-gradient momentum is dumped.

The weighting matrix  $A$  of Table 1 can be either an identity matrix or one of the mounting matrices, depending on the minimization criterion selected for the control law. For instance,  $A$  is the magnetic coil mounting matrix,  $M$ , for a minimum energy control law and is the reaction wheel mounting matrix,  $W$ , for a minimum wheel speed control law. For any orthogonal system,  $A$  is the identity matrix, and in the following discussion  $A$  is assumed to be the identity matrix.

The costate vector  $\vec{P}$  is defined differently for the closed-loop and open-loop control laws. For a closed-loop control law,  $\vec{P}$  is the desired torque weighted by  $|\vec{B}|^{-2}$ , where  $\vec{B}$  is



the geomagnetic field. For an open-loop control law,  $\vec{P}$  is the desired momentum weighted by both the magnitude and the direction of the geomagnetic field over the desaturation period. The physical meaning of  $\vec{P}$  for an open-loop control law is illustrated in Figure 2 with the assumption that the magnitude of  $\vec{B}$  is constant in time. As shown in Figure 2,  $\vec{P}$  is a fictitious torque whose component along the direction normal to the instantaneous geomagnetic field is the instantaneous magnetic torque,  $\vec{T}_M$ , generated by the torquers. The integration of  $\vec{T}_M$  over the desaturation period is equal to  $\vec{H}_D$ . The costate vector  $P$  in an open-loop control law is analogous to the desired torque  $\vec{T}_D$  in a closed-loop control law after being properly weighted.

Figure 2 also illustrates the significance of the desaturation period for an open-loop control law. Three cases covering different desaturation periods are shown in Figure 2. When the desaturation period is very short, as illustrated in Figure 2(a),  $\vec{P}$  approaches infinity due to the near-singular condition. In this case, the magnetic torquers are given poorly defined commands with the result that the magnetic torques generated may go through an undesirable path before the desired momentum is achieved. This is shown in Figure 2(a), where the magnetic torque  $\vec{T}_{M1}$  is along a direction almost opposite to the direction of the desired momentum  $\vec{H}_D$ . This can cause a very high reaction wheel speed at the end of  $t_1$ , which is undesirable. Thus, an open-loop control law operated under very short desaturation periods can sometimes lead to serious consequences. As the desaturation period increases as shown in Figure 2(b) and (c), the costate vector  $\vec{P}$  becomes better defined and the path of the magnetic torques becomes closer to the desired momentum.

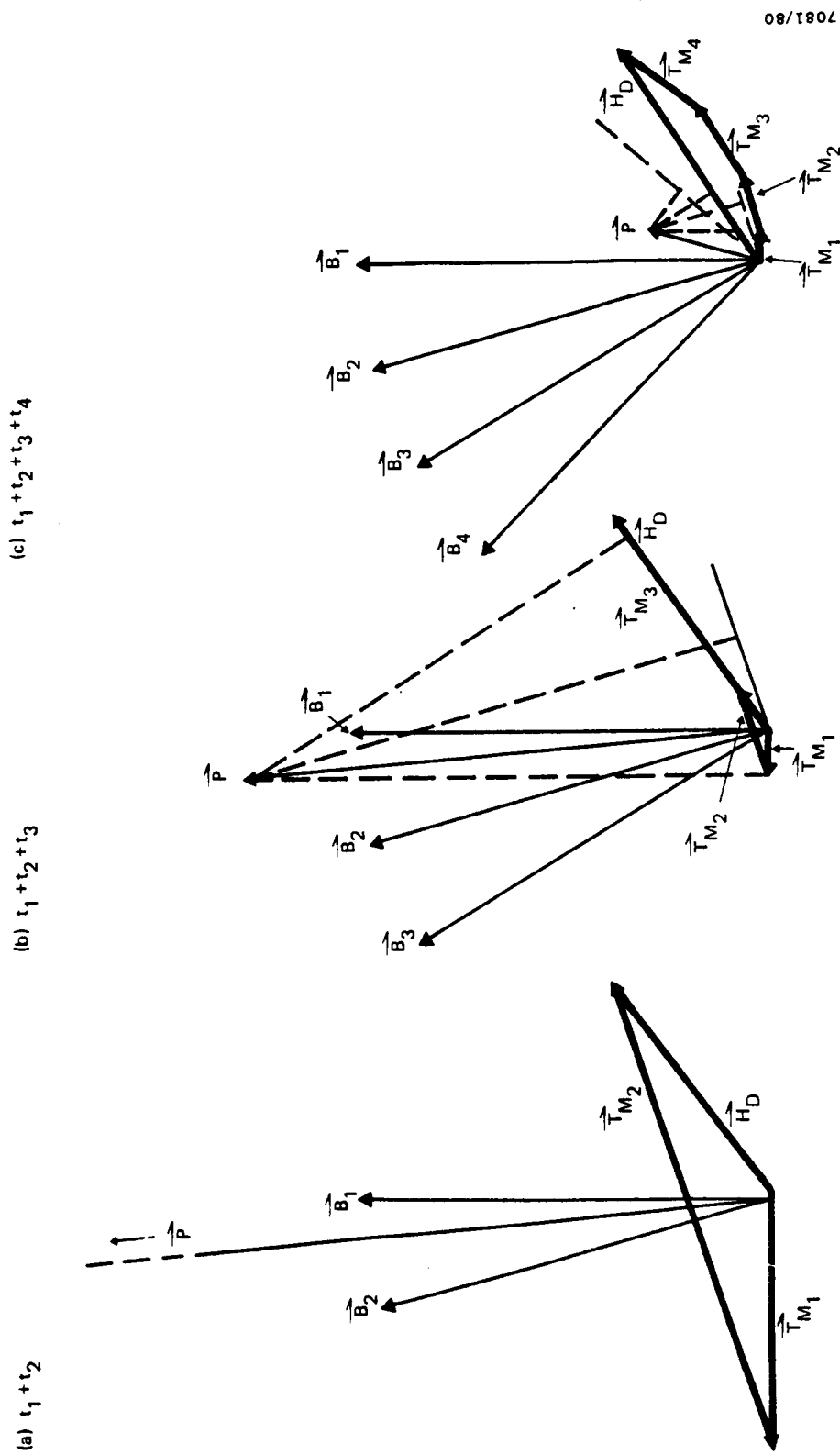


Figure 2. Physical Interpretation of the Costate Vector in an Open-Loop Control Law

Once the costate vector is determined, the remaining quantities of the control laws are computed through an identical set of equations for both the closed-loop and the open-loop control laws. The system magnetic dipole moment  $\vec{\mu}_M$  is the magnetic dipole moment (defined in the spacecraft body coordinate system) that is required to generate the desired magnetic torques. The magnetic torque  $\vec{T}_M$  is the actual instantaneous magnetic torque generated from the interaction between the magnetic torquers and the geomagnetic field. The magnitude and direction of  $\vec{T}_M$  are as follows. For a closed-loop control law,  $\vec{T}_M$  is the component of  $\vec{T}_D$  that is normal to  $\vec{B}$ . This component is the best torque that can be achieved because  $\vec{T}_M$  will be perpendicular to  $\vec{B}$ , although ideally it would be desirable to generate a  $\vec{T}_M$  that equals  $\vec{T}_D$ . Depending on the minimization criterion, when the weighting matrix  $A$  is different from the identity, the magnitude and direction of  $\vec{T}_M$  differ slightly from those described above. For an open-loop control law,  $\vec{T}_M$  is also perpendicular to  $\vec{B}$  at any moment. However, in this case  $\vec{T}_M$  also satisfies the condition that its integrated effect over the desaturation period equals the desired momentum,  $\vec{H}_D$ . That is,  $\vec{T}_M$  satisfies the condition that

$$\int_{t_i}^{t_f} \vec{T}_M dt = \vec{H}_D \quad (3)$$

This indicates that although the desired torques cannot always be generated momentarily, the desired momentum can usually be generated over a period of time, taking advantage of the variations in the geomagnetic field. This forms one major advantage of an open-loop control law over a closed-loop control law.

The last item in Table 1 is the commanded magnetic dipole moment  $\vec{\mu}_T$ . The components of  $\vec{\mu}_T$  give the actual dipole moment required by each of the magnetic torquers to generate the magnetic torque  $\vec{T}_M$ , and  $\vec{\mu}_T$  is the final output of a control law sent to the magnetic torquers.

### COMPARISONS AND DISCUSSIONS

Both the closed-loop and the open-loop control laws have their merits and drawbacks. The greatest problem of a closed-loop control law is that it attempts to achieve a desired torque momentarily, which is impossible in general. The closed-loop control law produces a magnetic torque that is the component of  $\vec{T}_D$  normal to the geomagnetic field. This effectively projects the resultant torque into the direction of the geomagnetic field, which is an unfavorable direction for further reduction of the momentum. As a result, a great deal of energy is wasted in changing the direction rather than reducing the magnitude of the momentum. Furthermore, the closed-loop control law attempts to always reduce the same fraction of the total momentum as controlled by the magnetic gain  $K_M$ , regardless of the variation in geometry. This is not efficient, because the law should always attempt to dump more momentum when the geometry is favorable and less momentum at an unfavorable geometry. In addition, the closed-loop control laws attempt to dump both the periodic and the nonperiodic gravity-gradient momenta, while only the nonperiodic portion needs to be dumped in most applications. These problems associated with the closed-loop control laws are eliminated in the open-loop control laws, because the open-loop control laws always look at the situation ahead of time to take advantage of the variations in geometry to dump the proper amount of momentum at the proper time. Thus, at the end of the desaturation

period, the exact amount of desired momentum will be generated from the torquers.

The open-loop control laws are ideal if actual performance is exactly as predicted. However, in case of modeling errors or undetected failure conditions, reality can be very different from the prediction. This difference will not be known until the end of the desaturation periods, which may be too late for correction. To resolve this potential problem, LMSC modified the open-loop control for ST so that a half-orbit desaturation period is used in computing the nominal momentum profile  $\vec{H}_{\text{NOM}}$  on the ground; then  $\vec{H}_{\text{NOM}}$  is used as the targeting momentum ( $\vec{H}_T(t_f)$  of Equation (2)) in computing  $\vec{H}_D$  on board where a much shorter desaturation period is used. With this modification, the advantages of the open-loop control laws are kept by forcing the system momentum to follow the same time variation it would follow if a half-orbit desaturation period were used under nominal situations. At the same time, the disadvantage of the open-loop control laws is reduced by decreasing the duration of the desaturation period so that the actual system momentum can be measured at a much higher frequency, and the deviation between the reality and the prediction can be included in  $\vec{H}_D$  and corrected for at this new frequency.

In principle, with a precomputed  $\vec{H}_{\text{NOM}}$ , the shorter the update period the better, if undetected failure conditions exist. However, as shown in Figure 2(a), making the desaturation period of an open-loop control law arbitrarily short may cause the costate vector  $P$  to be ill defined and result in very undesirable momentum before the desired momentum is achieved. For this reason, a 600-second desaturation period with a 200-second updating frequency was recommended in the current momentum management implementation for ST.

If instead of using an open-loop control law at a reduced desaturation period, a closed-loop control law is used with the precomputed  $\vec{H}_{NOM}$ , the problem of determining  $\vec{P}$  will no longer exist. In this technique, which we refer to as a mixed-mode control law, the updating frequency of  $\vec{P}$  can be reduced to the frequency of the closed-loop control laws, which is approximately 50 seconds for ST. To accomplish this, the desired torque at any time  $t$  will be computed with the following equation, which is directly obtained from Equation (1):

$$\vec{T}_D = \frac{1}{\Delta t} [\vec{H}_{NOM}(t + \Delta t) - \vec{H}_T(t)] - \vec{T}_{GG}(t) \quad (4)$$

where  $\Delta t$  is the updating frequency for the closed-loop control law and  $\vec{H}_{NOM}$  is the nominal momentum profile computed previously on the ground based upon an open-loop control law with a half-orbit desaturation period. The desired torque so determined is always nearly perpendicular to the instantaneous geomagnetic field because  $\vec{H}_{NOM}$  is computed from the nominal magnetic torques, which are momentarily perpendicular to  $B$ . This mixed-mode control law, which is a closed-loop control law operated with an open-loop  $\vec{H}_{NOM}$ , seems to retain the advantages of both the open-loop and the closed-loop control laws and is believed to be the best technique for momentum management. This mixed-mode control law is further described later in this paper.

### CURRENT ST IMPLEMENTATIONS

The current CP law implemented for ST is a closed-loop law that minimizes the reaction wheel speed. Thus, the desired torque  $\vec{T}_D$  and weighting matrix A of Table 1 are given by

$$\vec{T}_D = -K_M (\vec{H}_T + \vec{H}_B) - \vec{T}_{GG} \quad (5)$$

$$A = W = \begin{bmatrix} a & -a & a & -a \\ -b & -b & b & b \\ -b & -b & -b & -b \end{bmatrix} \quad (6)$$

where  $a = \sin 20$  degrees and  $b = 1/\sqrt{2} \cos 20$  degrees. The current ME law implemented for ST is a modified open-loop control law that minimizes the coil energy consumption. In this control law, a nominal momentum profile  $\vec{H}_{NOM}$  is computed on the ground for each of the inertial attitudes using a half-orbit as the desaturation period. This  $\vec{H}_{NOM}$  is then used in the determination of  $\vec{H}_D$  on board where a shorter desaturation period (600 seconds) and updating frequency (200 seconds) are used. As discussed earlier in the paper, the purpose of this modification is to reduce the error made in an open-loop control law in case undetected failure conditions exist. Thus, the desired momentum  $\vec{H}_D$  and the weighting matrix A of Table 1 are given by the following equations for the current ME law:

$$\vec{H}_D = \vec{H}_{NOM}(t_f) - \vec{H}_{RW}(t_i) - \int_{t_i}^{t_f} \vec{T}_{GG} \quad (7)$$

where  $t_i$  is updated every 200 seconds and  $t_f = t_i + 600$  seconds.

$$A = M = \begin{bmatrix} s & s & s & s \\ c & -c & -c & c \\ c & c & -c & c \end{bmatrix} \quad (8)$$

where  $s = \sin 35.26$  degrees and  $c = 1/\sqrt{2} \cos 35.26$  degrees.

Notice that in the determination of  $H_D$  for the current ME law, the total momentum  $\vec{H}_T$  given in Equation (1) has been replaced by the reaction wheel momentum  $\vec{H}_{RW}$ . This is due to the special way in which the current ME law is implemented, which does not require the knowledge of the system momentum during maneuvers. In the case of maneuvers, the normal mode of operation of the current ME law with a 600-second desaturation period and 200-second updating frequency is terminated. It is replaced by a single maneuver desaturation period that includes a lead time before the start of the maneuver and a lag time after the end of the maneuver. Thus, the length of the maneuver desaturation period depends on the lengths of the maneuver and the lead and lag times. In the current onboard implementation, each maneuver has a single lead/lag time that will be determined on the ground and uplinked to the spacecraft with the maneuver commands. This requires some ground software support in addition to the  $\vec{H}_{NOM}$  determination.

#### PROPOSED MIXED-MODE CONTROL LAW

As mentioned earlier in the paper, the mixed-mode control law, which retains the advantages of both closed-loop and



open-loop control laws, seems to be a good choice for momentum management using magnetic torquers. For the case of ST, the mixed-mode minimum wheel speed law, which is a closed-loop control law operated with an open-loop  $\vec{H}_{NOM}$  using the minimum wheel speed minimization criterion, would be optimal. In this case, the desired torque  $\vec{T}_D$  and the weighting matrix A of Table 1 are given by Equations (4) and (6), respectively. The advantages of this new technique over the current ST control laws are summarized below.

#### ADVANTAGES OVER THE CURRENT CP LAW

The mixed-mode minimum wheel speed law is better than the current CP law because it computes the desired torque based on the nominal momentum profile precomputed using an open-loop control law with a half-orbit desaturation period. The desired torque so determined has the following advantages:

1. It takes advantage of future geometrical variations so that the proper amount of momentum will be dumped at the proper time.
2. Only the nonperiodic portion of the gravity-gradient momentum will be dumped by the magnetic torquers.
3. The desired torque is always nearly perpendicular to the geomagnetic field so that very little energy will be wasted in changing the direction rather than reducing the magnitude of the momentum.
4. The reaction wheel center speed control loop is no longer needed because the targeting momentum  $\vec{H}_{NOM}$  automatically keeps the reaction wheel center speed at zero. This greatly simplifies the onboard computation.

### ADVANTAGES OVER THE CURRENT ME LAW

The mixed-mode minimum wheel speed control law has the following advantages over the current ME law:

1. It reduces the updating frequency of the costate vector  $P$  from 200 seconds to approximately 50 seconds. This will reduce the deviation between the actual and the predicted results when undetected failure conditions exist.
2. There is no need to define a desaturation period onboard. This eliminates the possibility of having a near-singularity condition in computing the costate vector  $P$ .
3. The required onboard computation is greatly simplified because it does not require the predicted geomagnetic field computation, and no integration is involved.
4. Minimization of wheel speeds reduces possible vibration in the spacecraft.

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